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ECONOMIES OF SCOPE IN RESEARCH AND DEVELOPMENT

BY

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Abstract

In this paper we derive the cost function for an innovator who completes two R.& D.-projects. We show then how this function includes the costs for two single project innovators as a special case. A comparison of the costs of innovation under these alternative R.&D.-organisations reveals that although the multiproject innovator benefits of intrafirm spillovers of knowledge between the projects, this is insufficient to conclude that he is less expensive. Therefore a static comparison of total knowledge necessary to complete different innovations leads toward the wrong conclusions regarding dynamic efficiency.

In view of the recent concern by antitrust authorities with predatory R.&D.-investment strategies, we finally look

at a sufficient condition for the existence of scope economies.

Acknowledgements

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1. INTRODUCTION

The involuntary dissemination of R.&D. results has been put forward recently for justifying public interventions to stimulate industries' innovative performance (Hartwick (1984), Spence (1984)). This inappropriability disincentive to engage in R.&D.-activities is an externality at the level of the market, i.e. it consists of knowledge overflows ("spillovers") between rival innovators. In this paper, dynamic efficiency of the private sector will be studied when such spillovers exclusively exist between research projects if completed by a single innovator. The question that emerges is whether a single multi-project innovator is less costly than several innovators each having just one project on the agenda. And the related public policy issue is whether a large enterprise engaged in several R.&D.-projects of which some turn out to be unprofitable does so for predatory motives, or simply because the knowledge generated by research on one (perhaps unprofitable) project can be used to some extent in future projects. Recent U.S. antitrust interventions against I.B.M. precisely adress this problem (Pittman (1984))[1].

In the paper we show that a simple comparison of total knowledge necessary to complete the different projects, as suggested by static intuition, is the wrong criterion for answering the cost-efficiency question. The reason is that when we account for diminishing returns to time compression (Scherer (1967), Mansfield (1968)), more knowlegde necessary to complete several projects can trigger diseconomies which outweigh the spillover advantage. The appropriate approach to the problem involves the application of the economies of scope concept to a multiproject R.&D.-cost function. The latter is obtained from a dynamic optimisation proces. With this analytical tool we can show that spillovers are a necessary but not a sufficient condition for economies of scope. Another condition then is put forward which under certain circumstances is sufficient for the existence of economies of scope.

2. THE MODEL

In this section we first present and discuss the features of the R.&D.-technology. Then we derive the costs for a multiproject innovator.

A. The R.&D.-technology

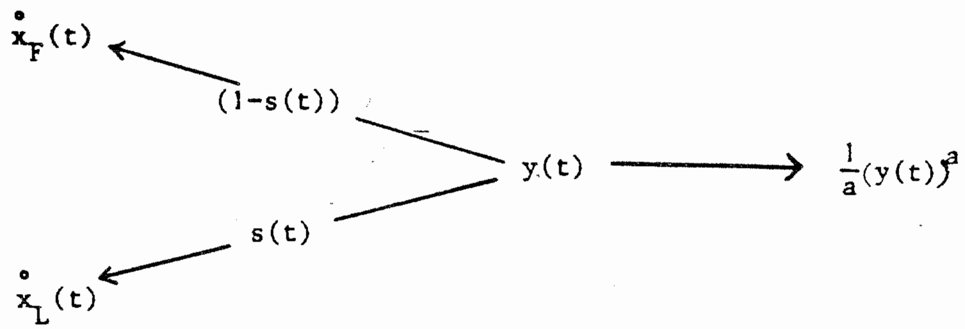
The innovator minimizes the discounted expenditures on the R.&D.-projects while accumulating enough knowledge to complete them by the resp. introduction dates T_1 and T_2. In order to have y units of knowledge, the innovator must spend $1/a\ y^a$ units of money. With $a > 1$ this amounts to the usual time compression diseconomies: if the innovator wishes to double the available knowledge at instant t, this more than doubles outlays that moment. Such an assumption most of the time underlies knowledge creation technologies (cfr. Kamien and Schwartz (1972), (1978), (1982); Lucas (1971); Vislie (1982a), (1982b), (1983)). The specific way to incorporate time compression diseconomies here is due to Reinganum (1982).

The innovator distributes the y units of knowledge over the projects on the agenda. The increase in the knowledge reached on the project with the latest completion date can be written as $\dot{x}_L(t) = s(t)\ y(t)$ where a dot denotes differentiation w.r.t. time, $x_L(\tau)$ stands for the cumulative knowledge reached on the latest project by time τ, and $s(t)$ stands for the share of y allocated to it at instant t. Similarly $\dot{x}_F(t) = (1 - s(t))\ y(t)$ gives the increase in knowledge for the first project.

Spillovers are taken into account in a very simple way: while in order to complete any project, an innovator has to accumulate an amount of knowledge A, in case he already has completed a project, this only amounts to $B < A$ for his latest project.

This R.&D.-proces can be schematically represented as follows:

Increase in knowledge for the projects at t	Total knowledge created in the lab.at t	outlays needed at t



In a more formal way, the cost function for the multiproject innovator satisfies:

$$\min_{y,\,s} \int_0^{T_L} e^{-rt}\,\frac{1}{a}(y(t))^a dt \qquad (1)$$

s.t. $\dot{x}_F(t) = (1-s(t))\,y(t)$ (2) $\quad x_F(0) = 0$ (3) $\quad x_F(T_F) = A$ (4)

s.t. $\dot{x}_L(t) = s(t)\,y(t)$ (5) $\quad x_L(0) = 0$ (6) $\quad x_L(T_L) = B$ (7)

s.t. $0 \leq s(t) \leq 1$ (8)

where in view of the argumentation above

- T_F is min $[T_1, T_2]$, the finishing date of the earliest project
- T_L is max $[T_1, T_2]$, the finishing date of the latest project
- $y(t)$ are the "units of knowledge" created at time t
- $s(t)$ is the share of this knowledge which is allocated to the project with the latest completion date
- $x_F(t)$ is the cumulative knowledge reached by the project with the first completion date by time t
- $x_L(t)$ is the cumulative knowledge reached by the project with the latest completion date by time t
- A is the total knowledge needed to obtain an innovation
- B is the total knowledge needed to obtain an innovation if the innovator already has completed a project. With positive spillovers: $B < A$.

a is the expenditure elasticity of knowledge, i.e. if knowledge has to increase by 1 %, monetary outlays increase by a %. With time compression diseconomies: $a > 1$.

r is the discount rate.

The R.&D.-technology represented by equations (1)-(8) assumes that the spillovers in question only can be valued if the same research entity (laboratory) develops also the latest project. There is no possibility to sell the relevant knowledge which is know-how locked-up in the laboratories' experience[2)]. Therefore the necessary conditions for the existence of economies of scope are satisfied:

"Economies of scope thus require both"the common and recurrent use of proprietary know-how or ... of a specialised and indivisible physical asset" (Teece, 1980, p. 223), and some element of market imperfection" (Olve (1985)).

But the same reason underlying this market imperfection--if consequently pursued--introduces an element counteracting the existence of scope economies into the picture. If knowledge spillovers consist of knowhow locked up in the research unit that completed the first project in such a way that it cannot be transferred to another laboratory by a market transaction, it is reasonable to assume that communication problems arise vis-a-vis additional researchers as well. Consequently the only way to cash upon the spillovers is to let the same crew develop both projects. If this implies an increase in the research intensity of the team, time compression diseconomies entail higher outlays. To put it simply, if investigators working in the lab. have to carry out additional assignments in order to finish the latest project in time, on top of their usual activities, overtime payments and other premiums increase more than proportionally the expenditures needed. Therefore we included time compression diseconomies at the level of the lab., not at the level of individual research projects. As will become clear below, this is the force counteracting scope. But first let us solve problem (1)-(8) for the multiproject innovator.

B. Solution

The multiproject innovator can follow either a sequential or a parallel R.&D.-strategy in order to introduce at T_1 and T_2. The cost of each option as well as the conditions under which each option will prevail optimally are summarised in proposition 1.

Proposition 1: a) The cost of parallel introduction at T_1 and T_2 is given by

$$C_{//}(T_1,T_2;A,B) = v(T_2-T_1)\left\{\frac{1}{a}\frac{(A+B)^a\, r^{a-1}}{\left((a-1)\left(e^{\frac{r}{a-1}T_2}-1\right)\right)^{a-1}}\right\}$$

$$+\, v(T_1-T_2)\left\{\frac{1}{a}\frac{(A+B)^a\, r^{a-1}}{\left((a-1)\left(e^{\frac{r}{a-1}T_1}-1\right)\right)^{a-1}}\right\} \qquad (9)$$

b) The cost of sequential introduction at T_1 and T_2 is given by

$$C_{\{\}}(T_1,T_2;A,B) = v(T_2-T_1)\left\{\frac{A^a\, r^{a-1}}{a\left((a-1)\left(e^{\frac{r}{a-1}T_1}-1\right)\right)^{a-1}} + \frac{B^a\, r^{a-1}}{a\left((a-1)\left(e^{\frac{r}{a-1}T_2}-e^{\frac{r}{a-1}T_1}\right)\right)^{a-1}}\right.$$

$$\left.+\, v(T_1-T_2)\left\{\frac{A^a\, r^{a-1}}{a\left((a-1)\left(e^{\frac{r}{a-1}T_2}-1\right)\right)^{a-1}} + \frac{B^a\, r^{a-1}}{a\left((a-1)\left(e^{\frac{r}{a-1}T_1}-e^{\frac{r}{a-1}T_2}\right)\right)^{a-1}}\right.\right.$$

$$(10)$$

c) A parallel strategy is followed whenever[3]

$$\frac{A}{B} < v(T_2-T_1)\,\frac{\left(e^{\frac{r}{a-1}T_1}-1\right)}{\left(e^{\frac{r}{a-1}T_2}-e^{\frac{r}{a-1}T_1}\right)} + v(T_1-T_2)\,\frac{\left(e^{\frac{r}{a-1}T_2}-1\right)}{\left(e^{\frac{r}{a-1}T_1}-e^{\frac{r}{a-1}T_2}\right)} \qquad (11)$$

where v is an indicator function satisfying

$$v(\text{arg.}) = 1 \text{ if arg.} > 0$$
$$= 0 \text{ if arg.} \leq 0$$

and // and {} are the symbols used to indicate a parallel resp. sequential strategy henceforth.

Proof:

The proof is the same regardless $T_1 \gtrless T_2$ and therefore is given in T_F and T_L. It gives the cost of both options as well as condition (11). Starting from equations (1) to (8), the problem of finding an optimal path from 0 to T_L is decomposed in finding an optimal path from 0 to T_F and an optimal path from T_F to T_L given that the initial conditions of the latter form the endpoint conditions of the first, and need to be determined in an optimal way. The strategy is to start with the problem of reaching T_L optimally assuming that one arrived already optimally in T_F (part a). This yields a cost function in the time remainder $T_L - T_F$, to acquire the remainder of knowledge B - S. Then we solve from 0 to T_F and include this cost function as a salvage value (part b). This enables us to find the optimal value of the state variable of the latest project at time T_F which then is used to obtain the final form of the cost enveloppe and to find condition (11).

part a:

The cost for adding B - S units of knowledge on the project with the latest completion date during the period $T_L - T_F$. It can be shown by straightforward application of the method proposed by Kamien and Schwartz (1972) that these costs are

$$C(T_L - T_F; B-S) = \frac{1}{a} (B-S)^a \left(\frac{r}{(a-1)\left(e^{\frac{r}{a-1} T_L} - e^{\frac{r}{a-1} T_F}\right)} \right)^{a-1} \tag{12}$$

In part b of the proof a detailed derivation of a similar result is given.

part b:

We now seek the costs for completing the first project, as well as already creating S units of knowledge for the latest project during the period $[0 - T_F]$, where S has to be determined optimally. This is tantamount to solve the program

$$\min_{y,s} \left(\int_0^{T_F} e^{-rt} \frac{1}{a} (y(t))^a \, dt + C(T_L - T_F \; ; B-S) \right) \tag{13}$$

s.t. $\dot{x}_F(t) = (1-s(t))\, y(t)$ (2) rep. $x_F(0) = 0$ (3) rep. $x_F(T_F) = A$ (4) rep.

s.t. $\dot{x}_L(t) = s(t)\, y(t)$ (5) rep. $x_L(0) = 0$ (6) rep. $x_L(T_F) = S$ (14)

s.t. $0 \leqslant s(t) \leqslant 1$ (10) rep.

We proceed by forming the Hamiltonian:

$$H \equiv -e^{-rt} \frac{1}{a} y^a(t) + \delta_F(t)\ (1-s(t))\ y(t) + \delta_L(t)\ s(t)\ y(t) \tag{15}$$

In view of condition (10), the associated Lagrangean is

$$L \equiv H + w_1\ (1 - s(t)) + w_2\ s(t) \tag{16}$$

where it holds that

$$w_1 \geq 0 \quad (17) \qquad w_2 \geq 0 \quad (18)$$

$$w_1\ (1 - s(t)) = 0 \quad (19) \qquad w_2\ s(t) = 0 \quad (20)$$

Then following conditions are both necessary and sufficient (Seierstad and Sydsaeter (1977)):

$$\frac{\partial L}{\partial y} = -e^{-rt}\ y^{a-1} + \delta_F(t)(1-s(t)) + \delta_L(t)\ s(t) \equiv 0 \tag{21}$$

$$\frac{\partial L}{\partial s} = -\ \delta_F(t)\ y(t) + \delta_L(t)\ y(t) - w_1 + w_2 \equiv 0 \tag{22}$$

$$-\ \mathring{\delta}_F(t) \equiv \frac{\partial L}{\partial x_F} = 0 \tag{23}$$

$$-\ \mathring{\delta}_L(t) \equiv \frac{\partial L}{\partial x_L} = 0 \tag{24}$$

Transversality conditions include:

$$\delta_L(T_F) \geqslant \frac{\partial C}{\partial S} = \frac{(B-S)^{a-1}\ r^{a-1}}{[\,(a-1)\,(\,e^{\frac{r}{a-1}T_L} - e^{\frac{r}{a-1}T_F})\,]^{\,a-1}} \tag{25}$$

$$S \geqslant 0 \qquad (26) \qquad\qquad (\delta_L + \frac{\partial C}{\partial S})\ S = 0 \qquad (27)$$

From (23) and (24) it is clear that the δ's are constants. Conditions (27) has an interesting economic interpretation: if the shadow cost of research for the latest project before the completion of the first exceeds the marginal cost of additional knowledge after completion (δ_L > $C/\ S$), the amount of knowledge reached on that project equals zero. Intuitively it is clear that it will not be optimal to spent on research before T_F if the resulting knowledge can be added more cheaply later on. Conversely, whenever there has been spending for research, we have (25) holding with equality, a condition that will be useful in the determination of the optimal S.

We now rewrite (22) as

$$(\delta_F - \delta_L)\ y(t) = w_1 - w_2 \tag{28}$$

Therefore, whenever $S > 0$, not only will δ_L be equal to $- C/S$, it will also equal δ_F for $S > 0$ implies $s > 0$ or $w_2 = 0$, together with $1 - s > 0$, a condition that always holds, implying $w_1 = 0$. As $y(t)$ is now zero, from (28) together with (27) we have

$$\delta_F = \delta_L = - C/S \qquad (29)$$

an important condition associated with parallel research ($S > 0$). For $S = 0$, or a sequential research strategy, we have $s = 0$ allowing $w_2 > 0$ or $\delta_F > \delta_L$. Together with (27) this results in

$$\delta_F > \delta_L > - C/S \qquad (30)$$

We will now distinguish according to these two cases of parallel and sequential research in solving the problem.

A. The parallel research strategy, or (29) holds.

First order condition (21) now can be written

$$- e^{-rt} y^{a-1} + \delta_L = 0 \qquad (31)$$

which enables us to obtain following expression for $y(t)$:

$$y(t) = \delta_L^{\frac{1}{a-1}} e^{\frac{r}{a-1} t} \qquad (32)$$

From the boundary conditions and the state equations (2), (3), (4), (5), (6) and (14) we have:

$$(A+S) = \int_0^{T_F} y(t)dt = \delta_L^{\frac{1}{a-1}} \frac{(a-1)}{r} (e^{\frac{r}{a-1} T_F} - 1) \tag{33}$$

which enables us to find an expression for δ_L and to eliminate this variable from (32). The expression for δ_L is useful in determining together with (25) the optimal value of S. This value is given by

$$S = \frac{(e^{\frac{r}{a-1} T_F} - 1)}{(e^{\frac{r}{a-1} T_L} - 1)} B - \frac{(e^{\frac{r}{a-1} T_L} - e^{\frac{r}{a-1} T_F})}{(e^{\frac{r}{a-1} T_L} - 1)} A \tag{34}$$

Eliminating δ_L from (32) and integrating yields the cost function as

$$C_{//}(T_F;A,S) = \frac{1}{a} (A+S)^a \left(\frac{r}{(a-1)(e^{\frac{r}{a-1} T_F} - 1)} \right)^{a-1} \tag{35}$$

Taking this expression together with expression (12) and substituting for the optimal value of S as given by (34) then gives the final expression for the costs of parallel research as

$$C_{//}(T_F, T_L; A, B) = \frac{1}{a} \frac{(A+B)^a \, r^{a-1}}{(a-1)^{a-1} \, (e^{\frac{r}{a-1} T_L} - 1)^{a-1}} \tag{36}$$

which proves part a) of proposition 1.

B. The sequential research strategy, or (30) holds.

As s = 0, first order condition (21) becomes

$$e^{-rt} \, y(t)^{a-1} + \delta_F = 0 \tag{37}$$

which enables us to obtain following expression for y(t):

$$y(t) = \delta_F^{\frac{1}{a-1}} \, e^{\frac{r}{a-1} t} \tag{38}$$

From the boundary conditions and the state equations (2), (3), (4) and given that s = 0, we have

$$A = \int_0^{T_F} (1-s(t)) \, \bar{y}(t) dt = \int_0^{T_F} y(t) dt = \delta_F^{\frac{1}{a-1}} \, \frac{(a-1)}{r} \, (e^{\frac{r}{a-1} T_F} - 1) \tag{39}$$

which enables us to find an expression for δ_F. Using this expression to eliminate δ_F in (38), and integrating gives the cost function as

$$C_{\{\}}(T_F;A,0) = \frac{1}{a} A^a \left(\frac{r}{(a-1)(e^{\frac{r}{a-1} T_F} - 1)} \right)^{a-1} \qquad (40)$$

Adding this expression to expression (12) in which we substitute S = 0 gives the final expression for the costs of sequential research as

$$C_{\{\}}(T_F,T_L;A,B) = \frac{1}{a} \frac{A^a \; r^{a-1}}{(a-1)^{a-1} \; (e^{\frac{r}{a-1} T_F} - 1)^{a-1}} + \frac{1}{a} \frac{B^a \; r^{a-1}}{(a-1)^{a-1} \; (e^{\frac{r}{a-1} T_L} - e^{\frac{r}{a-1} T_F})^{a-1}}$$

(41)

which proves part b) of proposition 1.

Now taking up the lead at equation (34), we see that

$$S > 0 \quad \text{iff} \quad \frac{(e^{\frac{r}{a-1} T_F} - 1)}{(e^{\frac{r}{a-1} T_L} - e^{\frac{r}{a-1} T_F})} > \frac{A}{B}$$

, (42) or (11) rep.

which proves part c) of proposition 1.

We now have an analytical tool describing the costs of multiproject innovation. The next issue we turn to is a comparison with the well-known single project costs.

3. ECONOMIES OF SCOPE: MULTIPROJECT VS. SINGLE PROJECT INNOVATORS

In this section we first show how the multiproject cost function derived encompasses the single project cost functions used previously in the literature. We then investigate under what conditions economies of scope are present in the underlying R.&D.-technology. Knowledge overflows between projects prove to be insufficient. Finally circumstances under which one can be sure of scope economies are put forward.

A. R.&D. by single project innovators

The cost for single project innovators can be obtained from the solution in proposition 1, by noting that an innovator not undertaking a project is equivalent to an infinite completion date for that project. By the finiteness of the introduction date of the project which remains on the agenda (this introduction date is T_F), the project which isn't done is the latest, and $T_L = \infty$. In such case the LHS of equation (42) will equal zero, so that the inequality is violated and the costs for the sequential option are to be used. At this point one ought to notice the usefulness of the two-branch formulation of the solution presented. As resp. the first, second project is dropped, we have resp. T_1, $T_2 = \infty$ and use resp. the second and first branch to plug in the infinite completion date. Application of this algorithm yields

$$C(T_1,\infty;A) = \frac{A^a \quad r^{a-1}}{a((a-1)(e^{\frac{r}{a-1}T_1} - 1))^{a-1}} \tag{43}$$

and

$$C(\infty,T_2;A) = \frac{A^a \quad r^{a-1}}{a((a-1)(e^{\frac{r}{a-1}T_2} - 1))^{a-1}} \tag{44}$$

which are the well-known single project cost functions for innovations of magnitude A at introduction dates T_1 and T_2. As a single project innovator has no "latest" project for which the accumulation of knowledge on the first project yields spillovers, total knowledge is each time A. This structure of the R.&D.-industry can not validate spillovers, the reduced knowledge B does not appear in the solution.

B. Spillovers and economies of scope

The just mentioned fact that spillovers don't exist between projects conducted by several innovators while they exist for a multiproject innovator results in less total knowledge needed to complete the projects when done by a multiproject innovator. Instead of 2 A, only A + B (remember B < A) has to be done. Can we conclude from this that the costs for joint research are lower? This question is equivalent to ask for the existence of economies of scope in the research technology. In order to answer it we redefine the scope economics concept to become operational for this intertemporal context as follows:

Definition 1. Economies of scope in R.&D. exist whenever

$$C(T_1,T_2;A,B) < C(T_1,\infty;A) + C(\infty,T_2;A) \qquad (45)$$

where $C(T_1,T_2;A,B) = \min [C_{\{\}}(T_1,T_2;A,B),\ C_{//}(T_1,T_2;A,B)]$

Notice that the idea in (45) is the same as the one behind the usual definition (Baumol, Panzar and Willig (1982)) but zero quantities of physical output there become infinite introduction dates for innovations in this context.

Proposition 2: Spillovers are a necessary but not a sufficient condition for economies of scope

Proof : As there are two options for the multiproject innovator, we have to distinguish according to the strategy he chooses.

For a sequential strategy the existence of scope implies (for $T_1 < T_2$):

$$\frac{B^a \quad r^{a-1}}{a((a-1)(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})^{a-1}} < \frac{A^a \quad r^{a-1}}{a((a-1)(e^{\frac{r}{a-1}T_2} - 1))^{a-1}} \tag{46}$$

$$\text{or} \quad \frac{B}{A} < \left(\frac{(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})}{(e^{\frac{r}{a-1}T_2} - 1)} \right)^{\frac{a-1}{a}} \tag{47}$$

Now $B/A < 1$ but also the RHS of (47) in view of $T_2 - T_1 < T_2$ has this property and hence spillovers are by no means a sufficient condition to guarantee scope, although clearly they are necessary.

For a parallel strategy the existence of scope implies (for $T_1 < T_2$):

$$\frac{1}{a}\frac{(A+B)^{a}\ r^{a-1}}{((a-1)(e^{\frac{r}{a-1}T_2}-1))^{a-1}} < \frac{1}{a}\frac{A^{a}\ r^{a-1}}{((a-1)(e^{\frac{r}{a-1}T_1}-1))^{a-1}} + \frac{1}{a}\frac{A^{a}\ r^{a-1}}{((a-1)(e^{\frac{r}{a-1}T_2}-1))^{a-1}} \tag{48}$$

or

$$\left(\frac{A+B}{A}\right)^{a} < \left(\frac{(e^{\frac{r}{a-1}T_2}-1)}{(e^{\frac{r}{a-1}T_1}-1)}\right)^{a-1} + 1 \tag{49}$$

Now B < A only implies that LHS (49) is smaller smaller than 2A but this does not prevent it from exceeding RHS (49) in which case no economies of scope prevail.Again (49) can be seen to be violated whenever B > A hence the necessity of spillovers.

What is the reason beyond this result? Suppose the multiproject innovator follows a sequential strategy.On the one hand he has an advantage in knowledge in comparison to the innovator completing the latest project, but on the other hand he has the disadvantage of having less time to complete the latest project.Time compression diseconomies prevalent in the $(T_2 - T_1)$ interval can destroy the positive externalities in knowledge. The immediate question arising is then why not turn to a parallel research strategy? This will leave less knowledge to be generated on the second project in the interval $(T_2 - T_1)$. But then more knowledge must result from research in the first period for in addition to the completion of the first project also the second has to be served. Again the prevailing time compression diseconomies will increase more than proportionally the necessary research outlays (which are closer in time and therefore more heavily discounted) so that the positive externalities might be outweighted. Therefore, in general, spillovers are by no means sufficient to induce economies of scope for in comparison with the single project innovators the multiproject innovator has a time availability disadvantage. Time compression diseconomies induced by the larger knowledge vis-a-vis each of the single project innovators taken separately then can outweigh the knowledge advantage that exists vis-a-vis the single project innovators taken together. This is basically what conditions (47) and (49) tell us: spillovers must be big enough (B small enough) to compensate for the smaller time remaining to complete a second project. In each of the conditions this emerges in a different way for each represents a different R.&D. strategy but one sees that both conditions are likely to be violated if the spillover advantage (A - B) is small (*too little*) and/or the finishing date for the second project shortly follows after the introduction date of the first project (*too late*).

C. A sufficient condition for economies of scope

As just argued, the mere fact that spillovers exist is not sufficient to conclude in favour of scope. What is needed is a proof of large enough spillovers. Needless to say that given the unobservable nature of these externalities this is difficult. Fortunately under certain circumstances the choice of strategy by the multiproject innovator himself will be sufficient to ensure the existence of scope. The idea is that spillovers will be big enough to induce scope if the completion date of the second project follows really shortly after the first introduction and still the innovator can complete the projects in a sequential way more cheaply than with a parallel strategy. This implies indeed that spillovers are sufficiently large so that no results for the second project must be generated before T_1 in view of too much compression diseconomies in the $[T_2 - T_1]$-interval, so that letting another innovator work during the entire time interval $[0 - T_2]$ solely on the second project will not save enough costs through time decompression to compensate for the spillover loss. How short the second date has to follow on the first depends among other things on the knowledge outlay elasticity and the discount rate, variables that can be estimated more easily.

To show this we start from the condition determining the choice of strategy for the multiproject innovator and ask when this conditions in strong enough to guarantee scope. Or given that a sequential strategy is followed and hence

$$\frac{B}{A} \leqslant \frac{(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})}{(e^{\frac{r}{a-1}T_1} - 1)} \tag{50}$$

when is this sufficient to induce

$$\frac{B}{A} < \left(\frac{(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})}{(e^{\frac{r}{a-1}T_2} - 1)} \right)^{\frac{a-1}{a}} \tag{51}$$

given the values of the parameters a and r? Clearly this occurs only if

$$\frac{(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})}{(e^{\frac{r}{a-1}T_1} - 1)} < \left(\frac{(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})}{(e^{\frac{r}{a-1}T_2} - 1)} \right)^{\frac{a-1}{a}} \tag{52}$$

which is equivalent to

$$(e^{\frac{r}{a-1}T_2} - e^{\frac{r}{a-1}T_1})(e^{\frac{r}{a-1}T_2} - 1)^{a-1} < (e^{\frac{r}{a-1}T_1} - 1)^{a} \tag{53}$$

Condition (53) is likely to be satisfied for research environments with introduction dates as depicted in the figure below

The question however is how close T_1 and T_2 have to be given the parameter values of a and r? For this purpose it is convenient to redefine $T_2 = bT_1$, where b then is the factor by which T_2 cannot exceed T_1 in order to be sure that a sequential strategy also implies scope economies. Making this substitution in (53) leaves us with still a fairly complex equation. As we assume $a > 1$ and further suppose that "a" takes on integer values, we have quadratic, cubic, quartic, ... equations, the roots of which yield values of "b" for which our sufficiency theorem holds. For the following table we have fixed "a" at 2 and then calculated the difference between T_2 and T_1 in months for different values of r. These are the elements of the table. The interpretation is thus:

for a discount rate of 10 %, a finishing date of the first project of 11 years and a second project finished 24 months later, and finally a sequential strategy followed, we can be sure of the existence of economies of scope. (If an entry is empty in this table, there are no positive values for which the sufficiency theorem holds).

Table 1. The number of months by which T_2 can exceed T_1 (expressed in years) for different discount rates (r) expressed as % on a yearly base.

	T_1							
r	5	7	9	11	13	15	17	19
5	-	-	-	-	-	-	19,0	34,6
7,5	-	-	-	9,4	25,0	33,0	35,6	34,7
8,75	-	-	3,5	20,6	28,6	30,5	29,0	25,9
10,00	-	-	13,7	24,0	26,7	25,2	22,3	18,6
14,00	-	13,1	19,2	18,2	14,9	11,7	9,1	7,1
16,75	3,5	15,8	15,8	12,6	9,5	7,0	5,2	3,8
19,5	8,7	14,2	11,8	8,6	6,1	4,3	3,0	2,1

For the computation of these values we applied discounting on a monthly basis, to approximate the continuous compounding in the analyses more closely. For any given introduction date of the first project, this table shows that an increase in the discount rate first relaxes the range in which T_2 has to fall in order to safely dismiss predatory R.&D., but beyond a point further increases in r constrain the safe range. Therefore one cannot simply generalise trends.

Although a decrease of the "safe" range with an increase of the discount rate has a nice intuitive appeal, for a higher discount rate makes a parallel strategy less attractive (monetary outlays spent early are more costly), the numerical analysis reveals that the intuition does not hold everywhere.

Further numerical analyses of equation (53) as well as an analyses of other sufficiency theorems certainly has to be done in view of antitrust policies. At the present however we showed that under certain circmustances a sequential strategy provides enough evidence in favour of scope economies. One however can not stress enough the sufficient nature of the condition: by no means can a parallel strategy be viewed as evidence pointing to the non-existence of scope economies. The only policy implication to be drawn from this research therefore is that predatory behavior can be dismissed in view of a sequential strategy under certain circumstances and with the underlying assumptions of the model holding true. Given the many extensions we will indicate as a matter of concluding this paper, a lot of future research remains to be done in order to use the above results in real world antitrust cares. It should however be clear that if anything is to be said on predatory R.&D. strategies, it has to come from an analysis of the above type. In view of the unobservable nature of the magnitude of the spillovers, one can only rely on conditions in which the innovators actions together with observable variables reveal enough. Of course, if one takes entrepreneurs' actions as rules of thumb, it might pay to adopt that kind of behaviour in order to avoid antitrust litigation. Future analyses also should take this into account.

CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

This paper has derived the costs for a multiproject innovator who benefitted from spillovers between the research projects. These showed to be insufficient to induce economies of scope. And the last condition itself is well-known to be insufficient for subadditivity of the cost function. Thus the existence of natural R.& D.-monopolies recquires more than just these often mentioned technological spillovers. What is needed is evidence of "large" spillovers, but how can this be established given the unobservable nature of the variable involved? In the paper we put forward conditions under which one could infer scope economies from an endogenous choice between different R.&D.-strategies followed by the innovator. Future research might set out for conditions from which one can infer subadditivity, and take into account that the magnitude of the spillovers might change with the option (sequential or parallel) chosen. In related research - however not focussing on the economies of scope issue - this has been the case (Van Cayseele (1985)).

Other extensions include a-symetric spillovers. At the present, spillovers imply that the project finished latest benefitted from the knowledge reached on behalf of an earlier project. With project related spillovers, however, knowledge overflows realise from working on a specific project (say the second). It then is obvious that a sequential strategy (for $T_2 > T_1$) results in no spillovers at all. Also under these circumstances the choice of strategy can reveal information on the existence of scope economies, but in a different way.

Finally, no technical uncertainty exists in the problem formulation of this paper. Once a certain amount of research has been reached the innovation is completed. Often however the only thing that can be said is that the probability of succes increases in the knowledge already reached on a project. The nature of the R.&D.-process then is stochastic. Spillovers now can have still another nature if f.i. the probability of having one succes affects the probability of a succes on a related project.

A final observation is related to positive rather than normative implications following from the above model. Recent empirical research by Aoki (1984) on what he calls the "hiving off" activities of large Japanese business groups during the late sixties and seventies revealed that parent firms do not specialise in "brain" activities or delegate only routine activities to the related firms. His empirical findings show that also R.& D.-activities have been hived off with "*wasteful* duplication of R.& D.-efforts in these multiple units" (Aoki (1984), p.193, (italics mine)).These results thus support the claims made in this paper. But we show how duplication is not (always) wasteful in a true economic sense. As certain real world dynamic features of the R.& D.-process such as time compression diseconomies are brought into the picture, the non-integrated solution follows out of minimum cost considerations. Of course, still other reasons (e.g. providing incentives through competition) might explain decentralised R.& D. out of optimal behaviour of the innovator. Therefore this research hopefully provides the framework for rethinking some existing analyses on innovative activities in terms of the theory of economies of scope.

NOTES

1. More precisely, Pittman (1984) says:
"In any industry in which technological change in rapid, the benefits of the development of a new product are likely to include improvements in other products sold by the firm and information which leads to the development of future products. These 'externalities' - external to the particular product line, internal to the firm - are likely to be difficult to quantify. Nevertheless, a rational firm must take them into account in its investment planning, and an analyst testing for predatation must include them in the benefits of a program".

2. In Nelsons' terminology (Nelson (1982)), the spillovers concerned are related to the "technique" part of technology (referring to a way of tackling problems in the pursuit of technology) which remains private rather than to the "logy" component (referring to the theoretical part of technology) that becomes public. If however knowledge overflows are of the easily disseminated know-why character of science, the market failures are even more obvious: why should someone pay for something he can obtain freely?

3. This is of course tantamount to

$$C(T_1,T_2;A,B) = \min [\ C_{\{\}}(T_1,T_2;A,B)\ ,\ C_{//}(T_1,T_2;A,B)]$$

Conditions (11) is however preferred over the above expression for it shows in the case of infinite introduction dates that the sequential option must be used to plus in ∞. For such cases the above expression would give a zero.

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